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12 Modeling of Power Systems for Small Signal Stability Analysis with FACTS

Small signal stability in a power system is the ability of the system to ascertain a stable operating condition following a small perturbation around its operating equilibrium. Power system disturbances can be broadly classified into two categories; large and small. Disturbances such as generation tripping, load outage, faults etc have severe influences on the system operation. These are large disturbances and the dynamic response and the stability conditions of the system are assessed within the standard framework of transient stability analysis and control. The system is modeled as a non-linear dynamic process. A large number of references dealing with this problem exist in power engineering literature [1]-[3]. Essentially the researchers have applied non-linear system theories and simulations to establish a clear understanding of the dynamic behavior of power system under such conditions. Effective tools to analyze and devise various non-linear control strategies are now in place.

The power system largely operates under quasi-equilibrium state except when undergoing large disturbance situations. The disturbances of small magnitude are very common. Such disturbances can come from the random fluctuation in loads induced by weather conditions etc. These small and gradual disturbances do not lead to severe excursion of system operating variables such as machine angle and speed from their operating equilibrium values. It is observed that the electromechanical oscillations observed in the post-fault recovery stage of the system are usually linear in nature [4]. The theory of linear system analysis has provided a deep insight into the operating behavior of an interconnected power system under such situations. The assumption of a linear system model around an operating equilibrium has revealed many interesting conclusions. Most often these conclusions are not consistent with what have been observed in the field under similar set of operating circumstances. A better understanding of the nature of the system dynamics helps to plan control strategies for secure operation.

This chapter will focus on modelling and analysis of power system dynamic behavior under small disturbances. A brief description of the modelling of various components in power systems including FACTS-devices is given. This chapter will focus on the dynamic model of FACTS-devices as their steady state power flow models have already been discussed in the earlier chapters. The overall system model is linearized for small signal stability analysis through eigenvalue approach. The small signal analysis will be applied in an interconnected power system model with FACTS-devices. The approach of modal controllability [4] will be described and applied to examine control capability of FACTS-devices from vari-

ous locations in the systems. We will also describe the methods of modal observability [4] to identify the most effective feedback signals for the control design of the FACTS-devices to produce greater stability margin. The aim of this chapter is

- to develop a clear understanding of how linear system theory can provide enhanced insight into power system dynamic behavior under various operating situations,
- to develop a better understanding of the control needs and specifications.

12.1 Small Signal Modeling

12.1.1 Synchronous Generators

The primary sources of electrical energy are the synchronous generators. They are electromechanical energy conversion devices that are driven at synchronous speed by steam, hydro and gas turbines, depending on the source of mechanical energy. The rotor houses a field winding which is excited through direct current to produce flux. The flux produces a rotational voltage in the stator windings which are connected to the grid. The magnitude of the voltage is controlled through an automatic voltage regulator (AVR). Power output is varied through controlled admission of steam or water or gas using a governor. The general approach to synchronous machine modelling is quite mature. The high frequency stator transient is usually ignored. Besides the field winding, the rotor might have a closed physical winding. The solid rotor body provides a closed rotor winding effect. At a speed other than synchronous, voltage is induced and currents circulate. They provide damping action against rotor speed deviation. Consequently the windings are known as damper winding. The rotor damping effect is modeled by closed windings of suitable inductances and time constants. The number of damper windings used to represent rotor damping effect depends on the nature of study. For small signal stability, two damper windings in the q-axis and one damper in the field axis are adequate. One can neglect the damper winding for model simplification at the cost of introducing some degree of conservatism in small signal stability results.

Let us assume an interconnected power system with m machine and n bus. We consider four windings on the rotor (one field and one damper in d-axis and two dampers in q-axis). For $i = 1$ to m , the following equations represent machine dynamics [5]. We have considered a d-q axis modeling of machine with the q-axis leading the d-axis and taken generator current as positive, i.e. IEEE convention [6] and [7].

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s \quad (12.1)$$

$$\frac{d\omega_i}{dt} = \frac{\omega_i}{2H_i} [T_{mi} - T_{elec i} - T_{Di}] \quad (12.2)$$

$$\begin{aligned} \frac{dE_{di}'}{dt} = & -\frac{1}{T_{qoi}} \left[+E_{di}' + (X_{qi} - X_{qi}') \right] \left[I_{qi} \left[-\frac{(X_{qi}' - X_{qi}'')}{(X_{qi}' - X_{lsi})^2} (-\Psi_{2qi} \right. \right. \\ & \left. \left. + (X_{qi}' - X_{lsi}) I_{qi} - E_{di}') \right] \right] \end{aligned} \quad (12.3)$$

$$\frac{d\Psi_{1di}}{dt} = \frac{1}{T_{doi}} \left[-\Psi_{1di} + E_{qi}' + (X_{di}' - X_{lsi}) I_{di} \right] \quad (12.4)$$

$$\frac{d\Psi_{2qi}}{dt} = -\frac{1}{T_{qoi}} \left[\Psi_{2qi} + E_{di}' - (X_{qi}' - X_{lsi}) I_{qi} \right] \quad (12.5)$$

$$\begin{aligned} T_{elec i} = & \frac{(X_{di}'' - X_{lsi})}{(X_{di}' - X_{lsi})} E_{qi}' I_{qi} + \frac{(X_{di}' - X_{di}'')}{(X_{di}' - X_{lsi})} \Psi_{1di} I_{qi} \\ & + \frac{(X_{qi}'' - X_{lsi})}{(X_{qi}' - X_{lsi})} E_{di}' I_{di} - \frac{(X_{qi}' - X_{qi}'')}{(X_{qi}' - X_{lsi})} \Psi_{2qi} I_{di} - (X_{qi}'' - X_{di}'') I_{qi} I_{di} \end{aligned} \quad (12.6)$$

$$T_D = D(\omega_s - \omega) \quad (12.7)$$

The stator current equations are algebraic in nature because of the assumption made earlier. They are:

$$V_i \cos(\delta_i - \theta_i) - \frac{(X_{di}'' - X_{lsi})}{(X_{di}' - X_{lsi})} E_{qi}' - \frac{(X_{di}' - X_{di}'')}{(X_{di}' - X_{lsi})} \Psi_{1di} + R_{si} I_{qi} - X_{di}'' I_{di} = 0 \quad (12.8)$$

$$V_i \sin(\delta_i - \theta_i) + \frac{(X_{qi}'' - X_{lsi})}{(X_{qi}' - X_{lsi})} E_{di}' - \frac{(X_{qi}' - X_{qi}'')}{(X_{qi}' - X_{lsi})} \Psi_{2qi} - X_{qi}'' I_{qi} - R_{si} I_{di} = 0 \quad (12.9)$$

where, for the i^{th} machine

- δ_i : rotor angle (radian)
- ω_i : rotor speed (radian per second)
- E_{fdi} : exciter voltage on stator base (p.u.)
- $E_{qi}'(E_{di}')$: quadrature (direct) axis transient voltage (p.u.).
- $\Psi_{1di}(\Psi_{1qi})$: flux linkage in the direct (inner quadrature) axis damper (p.u.)
- $I_{qi}(I_{di})$: stator q-axis(d-axis) component of currents (p.u.)

V_i, θ_i :	bus voltage magnitude and angle respectively.
X_{si} :	stator leakage reactance (p.u.)
R_{si} :	stator resistance (p.u.)
$X_{di}, X_{di}', X_{di}''$:	direct axis synchronous, transient and sub-transient reactance (p.u.) respectively.
$X_{qi}, X_{qi}', X_{qi}''$:	quadrature axis synchronous, transient and sub-transient reactance (p.u.) respectively.
T_{do}', T_{do}'' :	direct axis open circuit transient and sub-transient time constants (seconds) respectively.
T_{qo}', T_{qo}'' :	quadrature axis open circuit transient and sub-transient time constants (seconds) respectively

12.1.2 Excitation Systems

The excitation system provides the necessary rotor flux to induce a voltage in the stator. The excitation voltage E_{fdi} is never manipulated directly but is changed through the action of the exciter. Excitation systems are broadly classified into two types: slow DC excitation and fast static excitation [4].

A typical slow excitation system (termed DC1A exciter) [4] consists of four basic blocks as shown in Fig 12.1. They are the exciter, amplifier, excitation-stabilizer and terminal voltage sensor. A basic model for an exciter is given by equation (12.10) where S_e is the saturation in the exciter. It is approximated as an exponential function. The constants K_e and T_e relate to exciter gain and time constant respectively. The K_e varies with the operating conditions. For each operating condition, it is assumed that K_e is such as to make the voltage regulator output zero in the steady state. In order to automatically control the terminal voltage a measured voltage signal must be compared to a reference voltage and amplified to produce the exciter input, V_r . The amplifier can be a pilot exciter or a solid state amplifier. In either case, the amplifier is modeled as a first order differential equation as shown in equation (12.11). The regulator is often equipped with a stabilizing transformer that is modeled by equation (12.12).

The symbols K_a , T_a and K_f , T_f are the gain and time constant of the amplifier and stabilizer circuit respectively. The terminal voltage sensor is modeled as a first order block with a filter time constant T_r and shown in equation (12.13).

$$T_e \frac{dE_{fd}}{dt} = -[K_e E_{fd} + S_e(E_{fd}) E_{fd}] + V_r \quad (12.10)$$

$$T_a \frac{dV_r}{dt} = -V_r + K_a V_i \quad ; \quad V_{rmin} \leq V_r \leq V_{rmax} \quad (12.11)$$

$$T_f \frac{dR_f}{dt} = -R_f + \frac{K_f}{T_f} E_{fd} \quad (12.12)$$

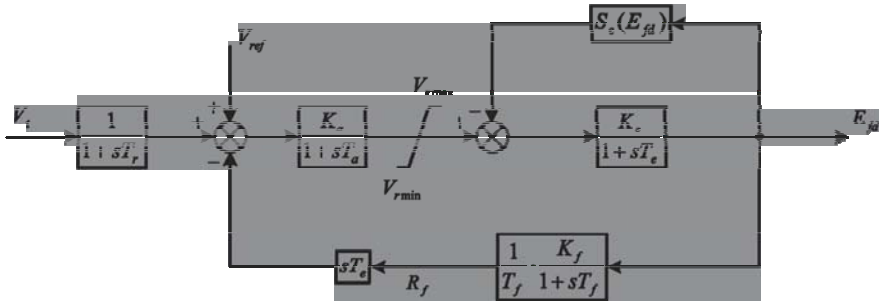


Fig. 12.1. Block Diagram of a DC1A-type Excitation System

$$\frac{dV_{tr}}{dt} = -\frac{1}{T_r} [V_{tr} - V_t] \quad (12.13)$$

Modern large machines are equipped with fast acting type Thyristor based excitation systems. The exciter power is drawn from the generator bus through an exciter transformer. Such a type of excitation system (ST1A) is often modeled as a single time-constant block [4]. The error signal is used as input and E_{fd} as output. Figure 12.2 shows a small signal representation for a high gain (of the order 200 to 400) and fast (of the order of a few milliseconds) exciter. Normally, T_a is neglected. When T_a is ignored, the dynamics are described by the following two equations:

$$\frac{dV_{tr}}{dt} = -\frac{1}{T_r} [V_{tr} - V_t] \quad (12.14)$$

$$E_{fd} = K_A (V_{ref} - V_{tr}) \quad (12.15)$$

In the case where the voltage regulator gain K_a is too large for better transient stability performance, the damping torque introduced by the exciter becomes negative. In order to ensure a well-damped post-fault response of the system, the regulator block is preceded by a transient gain reduction (TGR) block. However, with properly designed power system stabilizer (PSS) this block is not necessary.

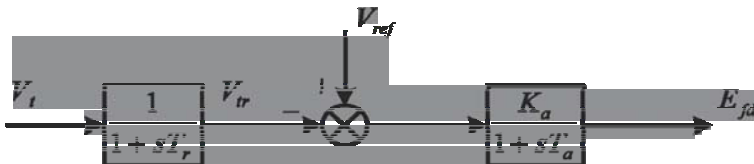


Fig. 12.2. Block Diagram of a Fast ST1A-type Excitation System

12.1.3 Turbine and Governor Model

In a power plant, the generator is driven either by a steam-turbine or a hydraulic turbine. Depending on the size, construction and principle of operation, different small signal models can be derived. For a steam turbine with tandem compound structure, various stages should be modeled adequately to represent the torsional dynamics [4]. The dynamic modeling of turbine and governor plays an important role in small signal stability studies. Interesting conclusions were drawn from a utility based field study in turbine and governor model validation. It was reported [8] that about 40% of simulated response could be observed during large generation trips in Western Electric Co-ordination Council (WECC) system. This allowed a response based modeling for the turbine governor system. The validated model produced a system response that matched closely the measured response of the system. Normally, for the electromechanical modes in the frequency range of 0.2 to 2.0 Hz, the dynamic interaction of these turbine masses can be an important consideration, if the associated governor is not properly tuned. All present day speed-governing systems are expected to be properly tuned, making the turbine less interactive. The inclusion of a small signal model of the turbine can increase the frequency of the low frequency electromechanical modes very slightly. As long as governors are properly tuned with adequate dead-band they will not have any adverse effect on power system damping. In view of this and for the sake of a simple model, the mechanical input to generator is assumed constant. However, for mid-term and long-term stability studies, which address system recovery from severe upsets with time, accurate modeling of turbine, governor is essential.

12.1.4 Load Model

In power system stability and power flow studies, the loads are modeled as seen from the bulk delivery point at transmission voltage level. Based on the way voltage and frequency influence loads at the delivery point, they are classified into two broad categories: static and dynamic.

In the static approach, both real and reactive loads are modeled as a non linear function of voltage magnitude. It also includes average frequency deviation (Δf). A static load model expresses the characteristics of the load at any instant of time as algebraic functions of the bus voltage magnitude and frequency at that instant. The active power component, P , and reactive power component, Q , are considered separately. The voltage dependency of the load characteristics is represented by the exponential model [4] as given in the following two equations.

$$P = P_0 (\bar{V})^a \quad (12.16)$$

$$Q = Q_0 (\bar{V})^b \quad (12.17)$$

$$\bar{V} = \left(\frac{V}{V_0} \right) \quad (12.18)$$

P_0 , Q_0 and V_0 are the values at the initial operating condition. The parameters of this model are the exponents 'a' and 'b'. With these exponents equal to 0, 1, 2, the model represents load of a constant power (CP), constant current (CC) or constant impedance (CI) type respectively. The exponent 'a' (or 'b') are sensitivities of power to voltage at $V = V_0$. For composite system loads, the exponent 'a' usually lies in the range between 0.5 and 1.8. Exponent 'b' varies as a non-linear function of the voltage. For Q at higher voltages, 'b' tends to be significantly higher than 'a'. An alternative model that has been widely used to represent the voltage dependency of loads is the polynomial model.

$$P = P_0 \left[p_1 (\bar{V})^2 + p_2 \bar{V} + p_3 \right] \quad (12.19)$$

$$Q = Q_0 \left[q_1 (\bar{V})^2 + q_2 \bar{V} + q_3 \right] \quad (12.20)$$

This model is commonly referred to as the ZIP model as it is composed of constant impedance Z , constant current I and constant power P components. The parameters of the model are the coefficients ' p_1 ' to ' p_3 ' and ' q_1 ' to ' q_2 ' that denote the proportion of each component. The frequency dependency of the load characteristic is usually represented in the exponential and polynomial models by a factor as follows:

$$P = P_0 (\bar{V})^a \left[1 + K_{pf} \Delta f \right] \quad (12.21)$$

$$Q = Q_0 (\bar{V})^b \left[1 + K_{qf} \Delta f \right] \quad (12.22)$$

$$P = P_0 \left[p_1 (\bar{V})^2 + p_2 \bar{V} + p_3 \right] \left(1 + K_{pf} \Delta f \right) \quad (12.23)$$

$$Q = Q_0 \left[q_1 (\bar{V})^2 + q_2 \bar{V} + q_3 \right] \left(1 + K_{qf} \Delta f \right) \quad (12.24)$$

Typically, K_{pf} ranges from 0 to 3.0 and K_{qf} ranges from -2.0 to 0.0.

Power system loads during a disturbance behave dynamically. However, because of the distributed nature of loads, it is difficult to get an equivalent dynamic representation of them. A large single induction motor load is modeled in the d-q reference frame almost in the same way as the synchronous generator. Some researchers represent loads through differential equations involving load voltage magnitude and angle as state variables. A power recovery model has been suggested in [9] for analyzing voltage stability related problems. It is shown that such models can capture voltage instability events more realistically.

The response of most of the composite loads to voltage and frequency changes is fast and the steady state condition for the response is reached very quickly. This

is true at least for modest changes in the voltage/frequency. The use of the static models described in the previous sections is justified in such cases.

There are, however, many cases where it is necessary to account for the dynamics of the load components. Studies of inter-area oscillations, voltage instability and long term stability often require load dynamics to be modeled. A study of systems with large concentrations of motors also requires the representation of load dynamics. Reference [4] discusses various models in use for stability studies and proposes a general model that encompasses a large variety of models with suitable modification of the coefficients. A CIGRE task force, formed to investigate the causes of the Swedish system blackouts in 1983, produced the following recommendations [10] on the effect of load models in stability studies in stressed power systems.

$$P_L = P_0 + P + K_{pw} \frac{d\theta_L}{dt} + K_{pv} \left(V_L + T \frac{dV_L}{dt} \right) \quad (12.25)$$

$$Q_L = Q_0 + Q + K_{qw} \frac{d\theta_L}{dt} + K_{qv1} V_L + K_{qv2} V_L^2 \quad (12.26)$$

Where, P and Q are static power loads, P_0 and Q_0 are the constant power portion of the induction motor load and the rest depends on bus voltage and frequency deviation. The symbols V_L and θ_L are the bus voltage magnitude and phase angle respectively. In our research we have simplified the load model shown in equation (12.25) and (12.26) according to:

$$P_L = P_{L0} \left(\frac{V_L}{V_{L0}} \right)^{np} + K_{pw} \frac{d\theta_L}{dt} + K_{pv} T \frac{dV_L}{dt} \quad (12.27)$$

$$Q_L = Q_{L0} \left(\frac{V_L}{V_{L0}} \right)^{nq} + K_{qw} \frac{d\theta_L}{dt} \quad (12.28)$$

12.1.5 Network and Power Flow Model

Power is transmitted over long distance through overhead lines of high voltage ranging from 230 kV to 1,100 kV. These overhead lines are classified according to length, based on the approximations used in their modeling:

- Short line: Lines shorter than 50 miles (80 km) are represented as equivalent series impedance. The shunt capacitance is neglected.
- Medium line: Lines, with length in the range of 80 km to about 200 km, are represented by nominal π equivalent circuits.
- Long Line: Lines longer than about 200 km fall in this category. For such lines the distributed effects of the parameters are significant. They need to be

represented by equivalent π circuits or alternatively as cascaded sections of shorter lengths, with each section represented by a nominal π equivalent.

For stability studies involving low frequency oscillations it is reasonable to assume a lumped parameter model. The approximation introduces a bit of conservatism in the margin of stability. However for simulation of lightning or switching transients, the distributed parameter model is used. High voltage transmission cables are also modeled in a similar way to overhead lines but they have much larger shunt capacitance than that of EHV lines of similar length and voltage rating.

In the steady state power frequency network model, the power flow equations at each node can be expressed as:

$$P_{G,k} - P_{L,k} = \sum_{m=1}^n V_k V_m (G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)) \quad (12.29)$$

$$Q_{G,k} - Q_{L,k} = \sum_{m=1}^n V_k V_m (G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)) \quad (12.30)$$

The symbols $P_{G,k}$ and $Q_{G,k}$ are real and reactive power generated respectively at the k^{th} bus. They are expressed as functions of bus voltage magnitude, angle and armature current as:

$$P_{G,k} = V_k \cos(\delta_k - \theta_k) I_{qk} - V_k \sin(\delta_k - \theta_k) I_{dk} \quad (12.31)$$

$$Q_{G,k} = -V_k \sin(\delta_k - \theta_k) I_{qk} - V_k \cos(\delta_k - \theta_k) I_{dk} \quad (12.32)$$

12.1.6 FACTS-Models

In this section, the steady-state and small-signal dynamic models of three most commonly used FACTS-devices are described. They are Static VAR Compensator (SVC), Controllable Phase Shifter (CPS) and Thyristor Controlled Series Capacitors (TCSC). We will mainly describe their steady state power flow characteristic and small signal dynamic characteristics. The power injection model has been used for steady-state representation of these devices as it is most suitable for incorporation into an existing power flow algorithm without altering the bus admittance matrix. The power injection equations governing this type of model are described for each of the devices. The small-signal dynamic models of the series connected devices are presented considering a single time constant block representing the response time of the power electronics based converters. For the shunt voltage control devices, a separate voltage control loop is involved with suitable response time of the voltage sensing hardware and time constants of the voltage regulator block.

12.1.6.1 SVC-Model

A commonly used topology of a Static VAR compensator (SVC), shown in Fig 12.3, comprises a parallel combination of a Thyristor Controlled Reactor and a fixed capacitor. It is basically a shunt connected static var generator/absorber whose output is adjusted to exchange capacitive or inductive current so as to maintain or control specific parameters of the electrical power system, typically bus voltage.

The reactive power injection of a SVC connected to bus k is given by

$$Q_k = V_k^2 B_{svc} \quad (12.33)$$

$B_{svc} = B_C - B_L$; the symbols B_C and B_L are the respective susceptances of the fixed capacitor and the Thyristor Controlled Reactor. It is also important to note that a SVC does not exchange real power with the system.

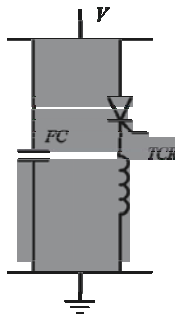


Fig. 12.3. SVC block diagram

The small-signal dynamic model of an SVC is given in Fig 12.4 [7]. ΔB_{svc} is defined as $\Delta B_C - \Delta B_L$. The differential equations from this block diagram can easily be derived as

$$\begin{aligned} \frac{d}{dt} \Delta B_{svc} = \frac{1}{T_{svc}} & \left[-\Delta B_{svc} + \left(1 - \frac{T_{v1}}{T_{v2}} \right) \Delta V_{r-svc} - \frac{K_v T_{v1}}{T_{v2}} \Delta V_{t-svc} \right] \\ & + \frac{K_v T_{v1}}{T_{v2} T_{svc}} [\Delta V_{ss-svc} + \Delta V_{ref}] \end{aligned} \quad (12.34)$$

$$\frac{d}{dt} \Delta V_{r-svc} = \frac{1}{T_{v2}} \left[-\Delta V_{r-svc} - K_v \Delta V_{t-svc} + K_v V_{ref} + K_v V_{ss-svc} \right] \quad (12.35)$$

$$\frac{d}{dt} \Delta V_{t-svc} = \frac{1}{T_m} [\Delta V_t - \Delta V_{t-svc}] \quad (12.36)$$

K_v , T_{v1} , T_{v2} are the gain and time constants of the voltage controller respectively; T_{svc} is the time constant associated with SVC response while T_m is the voltage sensing circuit time constant. The SVC can work either in voltage control mode or in susceptance control mode.

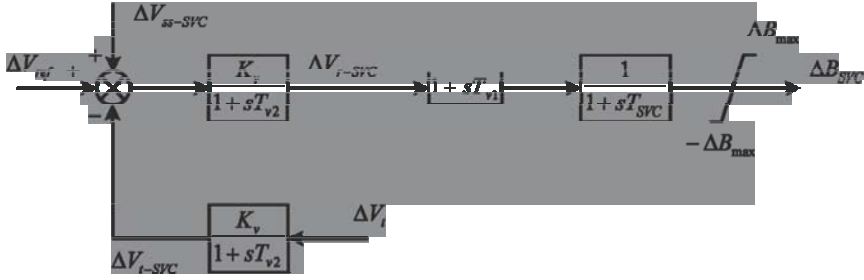


Fig. 12.4. SVC dynamic model

12.1.6.2 TCPS-Model

A Thyristor Controlled Phase Shifter (TCPS) can exert a continuous shift on the phase angle of voltage between the two ends of the line in which the TCPS is connected. A typical TCPS consists of an exciter and booster transformer pair. Fig 12.5 shows a typical TCPS connected in the line between bus k and m with its exciter transformer being fed by bus k. The injected voltage can be modeled as an ideal voltage source V_{se} in series with the line impedance Z_{km} . The injection model [11] is obtained by replacing the voltage source by an equivalent current source I_{se} in parallel with the line as shown in Fig 12.5 where I_{se} and I_{sh} are given by equation (12.37) and (12.38) and the injected power at both ends by (12.39) and (12.40).

$$\bar{I}_{se} = \frac{\bar{V}_{se}}{Z_{km}} \quad (12.37)$$

$$\bar{I}_{sh} = \bar{I}_k - \bar{I}_{se} \quad (12.38)$$

$$\bar{S}_k = \bar{V}_k (-\bar{I}_{sh} - \bar{I}_{se})^* \quad (12.39)$$

$$\bar{S}_m = \bar{V}_m (\bar{I}_{se})^* \quad (12.40)$$

The power injections S_k and S_m are given as

$$\bar{S}_k = \bar{V}_k (-\bar{I}_{sh} - \bar{I}_{se})^* \quad (12.41)$$

$$\bar{S}_m = \bar{V}_m (\bar{I}_{se})^* \quad (12.42)$$

The real and reactive components of the above two equations provide expression for nodal power injections which can be incorporated into network power flow equations. They are as follows:

$$P_{inj,k} = V_k V_m [G_{km} \{\cos \theta_{km} - \cos(\theta_{km} + \phi)\} + B_{km} \{\sin \theta_{km} - \sin(\theta_{km} + \phi)\}] \quad (12.43)$$

$$Q_{inj,k} = V_k V_m \left[G_{km} \{ \sin \theta_{km} - \sin(\theta_{km} + \phi) \} - B_{km} \{ \cos \theta_{km} - \cos(\theta_{km} + \phi) \} \right] \quad (12.44)$$

$$P_{inj,m} = V_m V_k \left[G_{mk} \{ \cos \theta_{mk} - \cos(\theta_{mk} - \phi) \} + B_{mk} \{ \sin \theta_{mk} - \sin(\theta_{mk} - \phi) \} \right] \quad (12.45)$$

$$Q_{inj,m} = V_m V_k \left[G_{mk} \{ \sin \theta_{mk} - \sin(\theta_{mk} - \phi) \} - B_{mk} \{ \cos \theta_{mk} - \cos(\theta_{mk} - \phi) \} \right] \quad (12.46)$$

where $\theta_{i,j} = \theta_i - \theta_j$.

The small-signal dynamic model of a controllable phase shifter is given in Fig 12.6. The small signal model is given by:

$$\frac{d}{dt} \Delta \phi = \frac{1}{T_{cps}} \left[-\Delta \phi + \Delta \phi_{ref} + \Delta \phi_{cps} \right] \quad (12.47)$$

The symbol T_{cps} represents the response time of the Thyristors.

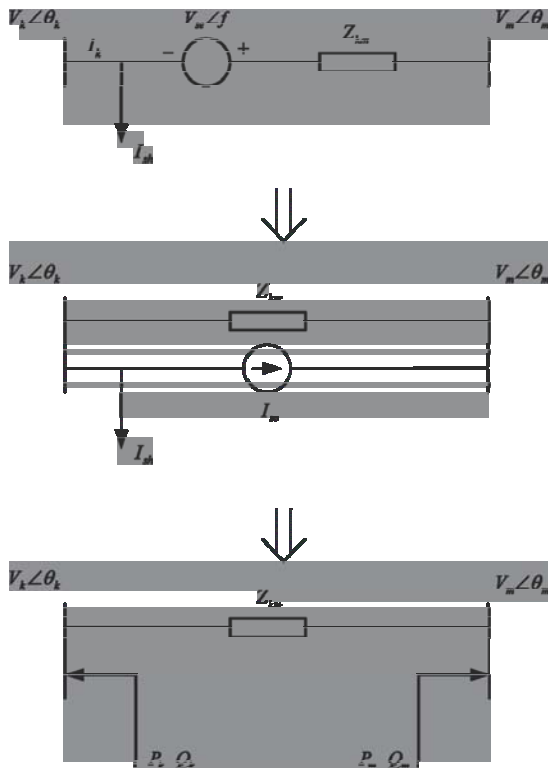


Fig. 12.5. TCPS block diagram

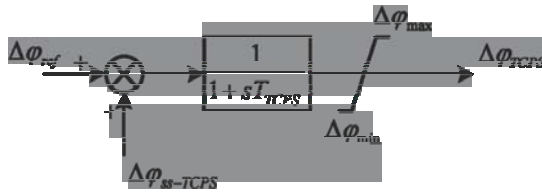


Fig. 12.6. TCPS dynamic model

12.1.6.3 TCSC-Model

A Thyristor Controlled Series Capacitor (TCSC) is a capacitive reactance compensator that consists of series capacitor banks shunted by Thyristor Controlled Reactors in order to provide a smoothly variable series capacitive reactance. Let us consider that the TCSC is connected in the line between bus k and m . In this case the resistance of the line is neglected for simplicity of the calculation. If I is the current flowing through the line, the TCSC having capacitive reactance X_c can be represented by a voltage source V_{se} as shown in Fig 12.7, where V_{se} is given by $V_{se} = jX_c I$. The injection model [11] is obtained by replacing the voltage source by an equivalent current source I_s in parallel with the line as shown in Fig 12.7 where I_s is given by $I_s = V_{se} / X_{km}$. The current source I_s corresponds to the injection powers S_k and S_m which are given by

$\bar{S}_k = \bar{V}_k (-I_s)^*$; $\bar{S}_m = \bar{V}_m (I_s)^*$. These expressions are resolved into real and reactive components to produce the following nodal power injection expressions:

$$P_k = \frac{k_c}{(k_c - 1)} V_k V_m B_{km} \sin(\theta_k - \theta_m) \quad (12.48)$$

$$Q_k = \frac{k_c}{(k_c - 1)} B_{km} [V_k^2 - V_k V_m \cos(\theta_k - \theta_m)] \quad (12.49)$$

$$P_m = \frac{k_c}{(k_c - 1)} V_m V_k B_{mk} \sin(\theta_m - \theta_k) \quad (12.50)$$

$$Q_m = \frac{k_c}{(k_c - 1)} B_{mk} [V_m^2 - V_m V_k \cos(\theta_m - \theta_k)] \quad (12.51)$$

where, $k_c = \frac{X_C}{X_L}$.

The small-signal dynamic model [7] of a controllable series capacitor is given in Fig. 12.8. The symbol T_{tcsc} represents the response time of the Thyristor. The model for various other FACTS-devices can be developed in similar manner.

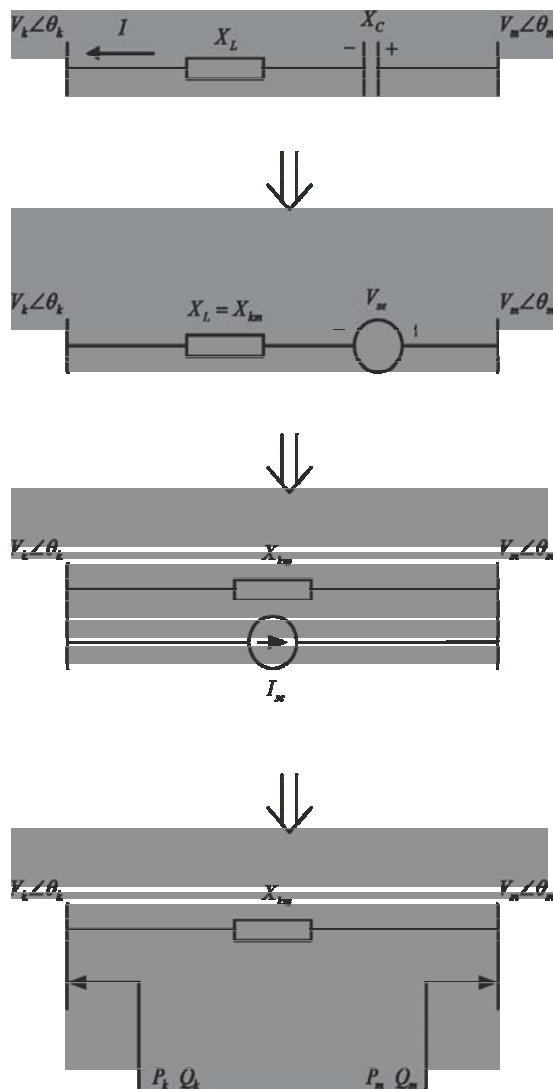


Fig. 12.7. TCPS block diagram

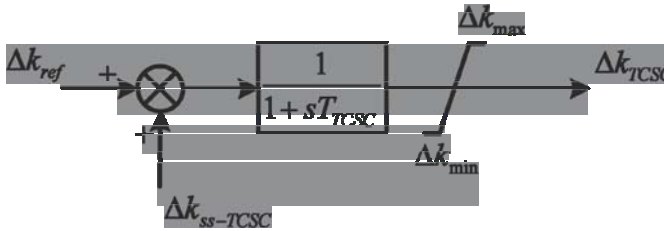


Fig. 12.8. TCPS dynamic model

12.1.7 Study System

The standard modeling approach described earlier will be applied to a study system. Fig. 12.9 shows a single line diagram of a 16-machine and 68-bus system model [7]. This is a reduced order equivalent of the New England Test System (NETS) and the New York Power System (NYPS) model. There are nine generators in NETS area and three in NYPS area. The three neighboring utilities are represented as three equivalent large generators #14, #15 and #16. The generators, loads and imports from other neighboring areas are representative of operating conditions in the early 1970s.

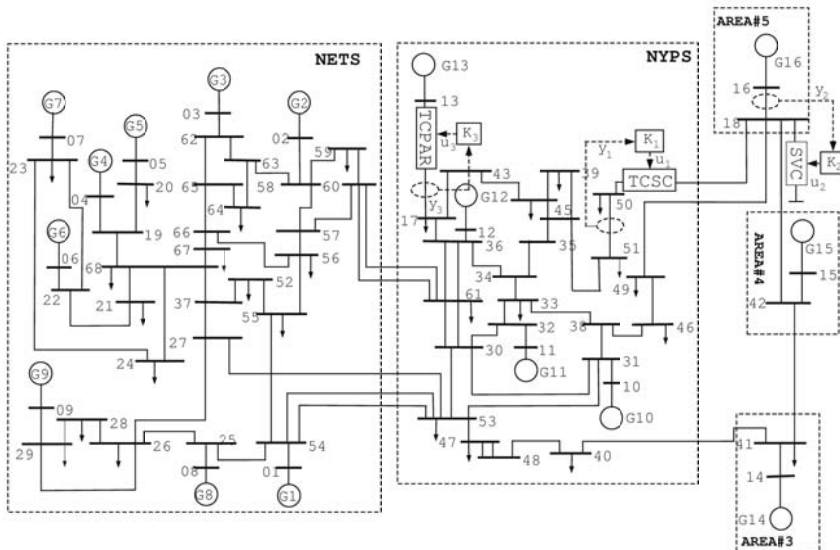


Fig. 12.9. 16-machine 68-bus study system

The first eight machines have slow excitation (IEEE type DC1A) whilst machine #9 is equipped with a fast acting static excitation system (IEEE ST1A). This machine is also assumed to have a speed input power system stabilizer (PSS) to ensure adequate damping of the electromechanical mode of this machine. The rest of the machines are under manual excitation control. We will analyze this system later for various modeling approximation and power flow and load characteristics to develop a better understanding of the system dynamics. We will also carry out the analysis with three FACTS-devices, SVC, TCSC and TCPS, in this system to see their influence on system dynamics.

12.2 Eigenvalue Analysis

12.2.1 Small Signal Stability Results of Study System

It is mentioned earlier that small signal stability is the ability of the system to maintain stable equilibrium when subjected to small disturbances. For small disturbances the response of the system will have linear behavior i.e. the equations that describe the resulting response of the system may be linearized for the purpose of analyses.

The behavior of a dynamic system such as a power system, as described in the earlier sections, can be expressed as a set of first order differential and algebraic (DAE) equations of the form:

$$\dot{x} = f(x, z, u) \quad (12.52)$$

$$0 = g(x, z, u) \quad (12.53)$$

$$y = h(x, z, u) \quad (12.54)$$

where, x is a state vector, z is an algebraic variable vector, u is an input vector and y is an output vector.

From a power system's perspective, the state vectors x are generator angle, speed, transient voltage, flux; excitation system voltage and AVR output etc. The algebraic variables z are bus voltage magnitude, angle, stator currents etc. The control variables u are excitation control reference voltage, mechanical input etc. The choice of output y variables depends on stabilizing signals such as line power, machine speed, bus voltage magnitude etc.

Let us linearize equations (12.52) to (12.54) around an initial (x_0, z_0, u_0) operating equilibrium and express the result as:

$$\Delta \dot{x} = \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial z} \Delta z + \frac{\partial f}{\partial u} \Delta u \quad (12.55)$$

$$0 = \frac{\partial g}{\partial x} \Delta x + \frac{\partial g}{\partial z} \Delta z + \frac{\partial g}{\partial u} \Delta u \quad (12.56)$$

$$\Delta y = \frac{\partial h}{\partial x} \Delta x + \frac{\partial h}{\partial z} \Delta z + \frac{\partial h}{\partial u} \Delta u \quad (12.57)$$

The algebraic variable Δz can be eliminated from the above to produce a state-space description of the system given by:

$$\Delta \dot{x} = A \Delta x + B \Delta u \quad (12.58)$$

$$\Delta y = C \Delta x + D \Delta u \quad (12.59)$$

where,

$$A = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial z} \left(\frac{\partial g}{\partial z} \right)^{-1} \frac{\partial g}{\partial x}; \quad B = \frac{\partial f}{\partial u} - \frac{\partial f}{\partial z} \left(\frac{\partial g}{\partial z} \right)^{-1} \frac{\partial g}{\partial u} \quad (12.60)$$

$$C = \frac{\partial h}{\partial x} - \frac{\partial h}{\partial z} \left(\frac{\partial g}{\partial z} \right)^{-1} \frac{\partial g}{\partial x}; \quad D = \frac{\partial h}{\partial u} - \frac{\partial h}{\partial z} \left(\frac{\partial g}{\partial z} \right)^{-1} \frac{\partial g}{\partial u} \quad (12.61)$$

The symbol Δ from equations (12.58) and (12.59) will be dropped in an effort to follow the notation of standard state space description. Unless stated otherwise, henceforth all perturbed variables will mean incremental variables. This approach of representing power system behavior in DAE form is largely followed for small signal analysis.

The eigenvalues λ of A are the roots of the characteristic equation:

$$\det(\lambda I - A) = 0 \quad (12.62)$$

Symmetric or Hermitian matrices will have real eigenvalues. On the other hand, non symmetric or non Hermitian matrices will have a few complex eigenvalues occurring in conjugate pairs. The complex conjugate eigenvalues are due to the fact that the matrix A is real and so the characteristic polynomial has real coefficients.

Let us take any complex conjugate eigenvalues $\lambda_{1,2} = \sigma \pm j\omega$. The real part σ relates to damping and the imaginary part ω relates to the frequency of oscillation. In power system small signal stability literatures, usually damping ratio ρ and linear frequency f (Hz) are used. These are related to λ_i as follows:

$$\lambda_i = \sigma_i \pm j\omega_i; \quad \rho_i = -\frac{\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}}; \quad f_i = \frac{\omega_i}{2\pi} \quad (12.63)$$

Let us use the classical model of all the generators with constant impedance load. In the classical model the airgap flux remains constant; so the effect of voltage regulator and damper circuit are absent. Only swing equations involving variables δ and ω are considered. The system will have thirty two state variables and so it will have as many eigenvalues. The eigenvalues are displayed in Table 12.1. There is one zero eigenvalue, one negative real and fifteen pairs of eigenvalues that occurs as complex conjugates. In this case the complex conjugate eigenvalues are known as electromechanical modes as they originate from the swing equations.

Table 12.1. Eigenvalues in classical model

λ_i	ρ_i	f_i (Hz)
0.0		
-0.1328	1.000	0.00
-0.0626±j2.4212	0.026	0.38
-0.0697±j3.1469	0.022	0.50
-0.0507±j3.9564	0.013	0.63
-0.0810±j4.9710	0.016	0.79
-0.0927±j6.1810	0.015	0.98
-0.0684±j6.7719	0.005	1.15
-0.0387±j7.2178	0.007	1.22
-0.0568±j7.6901	0.009	1.26
-0.0752±j7.9630	0.006	1.27
-0.0542±j7.9915	0.006	1.34
-0.0537±j8.4600	0.006	1.55
-0.0617±j9.7802	0.006	1.55
-0.0716±j9.7820	0.007	1.55
-0.0701±j9.8349	0.007	1.56
-0.1161±j11.467	0.010	1.82

We first explain the origin of the zero eigenvalue. The machine speeds and angles are expressed in absolute terms thereby introducing redundancies in the state variables and the resulting state matrix is singular. The zero eigenvalue is because of redundancy in angle but this can be removed by taking one machine angle as a reference and expressing all other angles with respect to it. This will result in reduction of angle state variable of the reference machine from the differential equations. Sometimes a second zero eigenvalue can exist when the generator torque is independent of machine speed deviations, i.e. mechanical damping is neglected and governor action is not represented. Because non-uniform damping is used, the second zero eigenvalue in this case does not exist. This situation can also arise when the ratios of inertia constant to damping coefficient in all the machines are uniform which can be avoided by setting the speed of one machine as reference (following the assumption of infinite inertia of a speed referenced machine) and expressing speed deviation of other machines with respect to the reference one. If one particular machine angle and speed are taken as reference, the dynamics of that particular machine will not affect the swing equation. In practice it is not done, as this introduces difficulties in indexing and manipulating various matrices and vectors in vector based computation. Usually the eigenvalues will not be exactly zero as initial conditions are not exact because of mismatches in power flow convergence, however small they might be.

The eigenvalues characterised by frequencies 0.38, 0.50, 0.63 and 0.79 Hz are known as inter-area modes [4] involving machines across a large portion of the system. The other eigenvalues in the table are local modes, involving one or two machines and hence the effect is localised.

We now show eigenvalue analysis results for detailed machine models, both with and without FACTS-devices, and for different network configuration and load situations. The eigenvalues for the system using full models are computed

and displayed in Table 12.2. Each machine is modelled to have three damper windings, one field winding and an excitation control system. The first eight generators use DC excitation, while machine #9 is equipped with fast excitation. The load is constant impedance in nature. Other machines are placed on manual excitation control. One can see that one local mode is unstable, which is connected to machine #9. This is due to a fast excitation control system in that machine. This mode is stabilised with the help of a speed input PSS. The behaviour of the system with the PSS is also shown in Table 12.2. It can be observed readily that the inclusion of damper windings in the model has increased the damping of the electromechanical modes in general. The effect of excitation control is also seen to have improved the frequencies of oscillations.

Table 12.2. Electromechanical modes in detailed model

Detailed model without PSS		Detailed model with PSS	
ρ_i	f_i (Hz)	ρ_i	f_i (Hz)
0.0165	0.3916	0.0643	0.3830
0.0436	0.5022	0.0436	0.5019
0.0345	0.6263	0.0560	0.6193
0.0498	0.7907	0.0499	0.7907
0.0627	1.0710	0.3061	0.8539
0.0578	1.1583	0.0630	1.0707
-0.0043	1.1895	0.0589	1.1584
0.0793	1.2050	0.0798	1.2045
0.0743	1.2716	0.0574	1.2640
0.0070	1.2951	0.0745	1.2718
0.0349	1.3516	0.0502	1.3418
0.0976	1.5400	0.0977	1.5400
0.0690	1.5455	0.0681	1.5470
0.0906	1.5639	0.0907	1.5637
0.0615	1.8760	0.0616	1.8759

The effect of FACTS on the damping of electromechanical modes is also investigated. We assume TCPS, TCSC and SVC are located in the network to facilitate power flow and provide network voltage support. The TCPS is assumed to be installed in the line between bus 37 and bus 68, the TCSC between bus 69 and bus 50 and the SVC is located at bus 18. The effect of each of these devices is investigated separately. The results are displayed in Table 12.3. It can be seen that the steady state outputs from these FACTS-devices do not improve the damping. We also include three devices and computed their combined effect in system damping. The observation was that the overall system damping did not improve appreciably. This means additional control known as supplementary power oscillation damping is necessary for improved system response. We now examine the effect of various levels of power flow on the damping of these electromechanical modes. We assume a 700 MW flow between NETS and NYPS as base and adjust the load and generation in both the areas to create a flow that varies from 100 MW to 900 MW.

Table 12.3. Effects of FACTS on electromechanical modes

No FACTS		SVC (only)		TCSC (only)		TCPS(only)	
ρ_i	f_i (Hz)	ρ_i	f_i (Hz)	ρ_i	f_i (Hz)	ρ_i	f_i (Hz)
0.06	0.38	0.06	0.38	0.06	0.39	0.06	0.38
0.04	0.50	0.04	0.50	0.04	0.50	0.04	0.50
0.05	0.62	0.05	0.62	0.05	0.62	0.05	0.62
0.05	0.79	0.05	0.79	0.05	0.79	0.05	0.79
0.30	0.85	0.30	0.85	0.30	0.85	0.30	0.85
0.06	1.07	0.06	1.07	0.06	1.07	0.06	1.07
0.06	1.16	0.06	1.16	0.06	1.16	0.06	1.16
0.08	1.20	0.08	1.20	0.08	1.20	0.08	1.20
0.06	1.26	0.06	1.26	0.05	1.26	0.05	1.26
0.07	1.27	0.07	1.27	0.07	1.27	0.07	1.27
0.05	1.34	0.05	1.34	0.05	1.34	0.05	1.34
0.10	1.54	0.10	1.54	0.09	1.54	0.10	1.54
0.07	1.54	0.07	1.54	0.07	1.54	0.06	1.54
0.09	1.56	0.09	1.56	0.09	1.56	0.09	1.56
0.06	1.87	0.06	1.87	0.06	1.87	0.06	1.87

The results are shown in Table 12.4. It is seen that at higher level of power flow, the damping and frequencies of the first inter-area mode reduces. The other modes do not show much change because they are not affected by the power flow between these two areas but instead are affected by flows between other areas.

This is owing to the reduced voltage at the two ends, which is picked up by the AVR in each area. The degradation of damping is not much because of the fact that only one area (NETS) has slow excitation control and the other area (NYPS) is on manual excitation control.

Table 12.4. Effects of power flow on electromechanical modes

100 MW		500 MW		700 MW		900 MW	
ρ_i	f_i (Hz)	ρ_i	f_i (Hz)	ρ_i	f_i (Hz)	ρ_i	f_i (Hz)
0.07	0.39	0.07	0.38	0.06	0.38	0.06	0.37
0.04	0.50	0.04	0.50	0.04	0.50	0.04	0.50
0.06	0.64	0.06	0.63	0.05	0.62	0.05	0.60
0.05	0.79	0.05	0.79	0.05	0.79	0.04	0.79
0.30	0.85	0.30	0.85	0.30	0.85	0.30	0.85
0.06	1.07	0.06	1.07	0.06	1.07	0.06	1.06
0.06	1.15	0.06	1.15	0.06	1.16	0.06	1.16
0.08	1.20	0.08	1.20	0.08	1.20	0.08	1.20
0.06	1.26	0.06	1.26	0.05	1.26	0.05	1.26
0.07	1.28	0.07	1.27	0.07	1.27	0.07	1.26
0.05	1.34	0.05	1.34	0.05	1.34	0.05	1.34
0.10	1.54	0.10	1.54	0.09	1.54	0.10	1.54
0.07	1.54	0.07	1.54	0.07	1.54	0.06	1.54
0.09	1.56	0.09	1.56	0.09	1.56	0.09	1.56
0.06	1.87	0.06	1.87	0.06	1.87	0.06	1.87

Usually fast excitation control significantly reduces the damping of low frequency modes. The frequency also reduces slightly with power flow because of reduced synchronising power co-efficient at relatively high rotor angles.

We now investigate the effect of load characteristics on the damping. Table 12.5 displays the results. So far in all our calculations, constant impedance (CI) loads are assumed. We have investigated the effect of various load characteristics such as constant current (CC), constant power (CP) and dynamic load at a few buses. The results in Table 12.5 show that the damping action from the constant power type of load is least. This is why the voltage and angle stability margin involving constant power type of load is low. The induction motor type load on the other hand produces better damping because they are asynchronous in nature. It is very difficult to quantify the effect of loads on damping. It is system specific and depends on the relative locations of loads, generation and tie lines in the system. The effect of tie line strength on system damping is investigated next and the results are displayed in Table 12.6.

We assume 700 MW flows with different tie line strength. The base case is with all ties in operation. One line between bus 53 and 54 connecting NETS with NYPS is then taken out. The eigenvalue analysis shows that the damping of the first three inter-area modes is reduced. In addition, if one line between bus 60 and 61 is taken out, the damping is reduced further. This quantitatively confirms our simple understanding that the power system with weak tie-line strength experiences oscillations. The mechanism of reduction in damping can be attributed to higher angular separation between the two areas. The maximum power transfer capacity reduces because of high transmission impedance. This demands increase in angular separation between two areas for the same amount of power flow. As the voltages at different buses are reduced; the overall operating situation leads to reduced damping and frequency of oscillations.

Table 12.5. Effect of load characteristics on electromechanical modes

CC		CP		CI		Dynamic	
ρ_i	f_i (Hz)	ρ_i	f_i (Hz)	ρ_i	f_i (Hz)	ρ_i	f_i (Hz)
0.05	0.38	0.05	0.36	0.06	0.38	0.07	0.38
0.04	0.51	0.04	0.53	0.04	0.50	0.04	0.50
0.06	0.62	0.06	0.64	0.05	0.62	0.07	0.62
0.05	0.79	0.05	0.79	0.05	0.79	0.04	0.79
0.30	0.88	0.29	0.91	0.30	0.85	0.30	0.85
0.06	1.07	0.06	1.07	0.06	1.07	0.06	1.07
0.06	1.15	0.06	1.15	0.06	1.16	0.06	1.16
0.08	1.20	0.08	1.20	0.08	1.20	0.08	1.20
0.06	1.26	0.06	1.26	0.05	1.26	0.06	1.26
0.07	1.27	0.07	1.27	0.07	1.27	0.07	1.27
0.05	1.34	0.05	1.34	0.05	1.34	0.05	1.34
0.10	1.54	0.10	1.54	0.09	1.54	0.10	1.54
0.07	1.54	0.07	1.54	0.07	1.54	0.07	1.54
0.09	1.56	0.09	1.56	0.09	1.56	0.09	1.56
0.06	1.87	0.06	1.87	0.06	1.87	0.06	1.87

Table 12.6. Effect of tie-line strength on electromechanical modes

No outage		Line 53-54 out		Line 60-61, 53-54 out	
ρ_i	f_i (Hz)	ρ_i	f_i (Hz)	ρ_i	f_i (Hz)
0.06	0.38	0.05	0.36	0.04	0.36
0.04	0.50	0.04	0.50	0.04	0.50
0.05	0.62	0.04	0.59	0.04	0.57
0.05	0.79	0.05	0.79	0.05	0.79
0.30	0.85	0.30	0.85	0.30	0.85
0.06	1.07	0.06	1.07	0.06	1.05
0.06	1.16	0.06	1.15	0.06	1.15
0.08	1.20	0.08	1.20	0.08	1.20
0.05	1.26	0.06	1.25	0.06	1.26
0.07	1.27	0.07	1.27	0.07	1.27
0.05	1.34	0.04	1.31	0.05	1.34
0.09	1.54	0.10	1.54	0.10	1.54
0.07	1.54	0.07	1.54	0.07	1.54
0.09	1.56	0.09	1.56	0.09	1.56
0.06	1.87	0.06	1.87	0.06	1.87

12.2.2 Eigenvector, Mode Shape and Participation Factor

If λ_i is an eigenvalue of A , v_i and w_i are non zero column and row vectors respectively such that the following relations hold:

$$(A - \lambda_i I)v_i = 0 \quad (12.64)$$

$$w_i(A - \lambda_i I) = 0 \quad (12.65)$$

The vectors v_i and w_i are known as right and left eigenvectors of matrix A . In a matrix with all distinct eigenvalues (not strictly necessary) one can arrange all eigenvectors and eigenvalues through compact matrix notations such as:

$$AV = V\Lambda \quad (12.66)$$

$$WA = \Lambda W \quad (12.67)$$

where,

$$V = (v_1 \quad v_2 \quad \dots \quad v_{n-1} \quad v_n) \quad (12.68)$$

$$W = (w_1^t \quad w_2^t \quad \dots \quad w_{n-1}^t \quad w_n^t)^t \quad (12.69)$$

$$\Lambda = \text{diag}(\lambda_1 \quad \lambda_2 \quad \dots \quad \lambda_{n-1} \quad \lambda_n) \quad (12.70)$$

The eigenvector matrices can be used as transformation matrices to transform the state variables x into decoupled modal variables z_m . The advantage of this transformation is that these variables are decoupled. The time domain behaviour of each of them completely represents the contribution of a particular eigenvalue (λ_i)

to overall system response. Pre-multiplying (12.66) by V^T and (12.67) by W^T modal matrix Λ can be obtained. One can transform physical state variables x into modal variables z with the help of eigenvector matrices V and W as follows:

$$x = Vz \quad (12.71)$$

$$z = Wx \quad (12.72)$$

The right eigenvector (v_i) is known as mode shape corresponding to λ_i . The mode shape is very useful in identifying a group of coherent generators in a multi-machine system. We compute right eigenvector corresponding to eigenvalue $0.0626 \pm j2.4212$ in Table 12.1. The entries in the eigenvector corresponding to machine speed are shown in Fig. 12.10. This shows two clusters of generators oscillating against each other. This is very important information for devising control strategies.

Of the fifteen complex conjugate eigenvalues, the first four when ranked in ascending order of frequency are known as the inter-area modes. In a large power system, it is important to quantify the role of any particular generator in one particular mode. This helps to simplify the dynamic characterization of the entire system by the reduced dynamic model to make the analysis and control synthesis much easier. It is natural to suggest that the significant state variables influencing a particular mode are those having large entries corresponding to the right eigenvector of λ_i . The problem of entries in an eigenvector is that they can not be compared with each other because they have different units and scaling i.e. entries in the eigenvector corresponding to state variables such as speed, angle, flux, voltage etc. can not be compared. Let us take a closer look at equations (12.71) and (12.72).

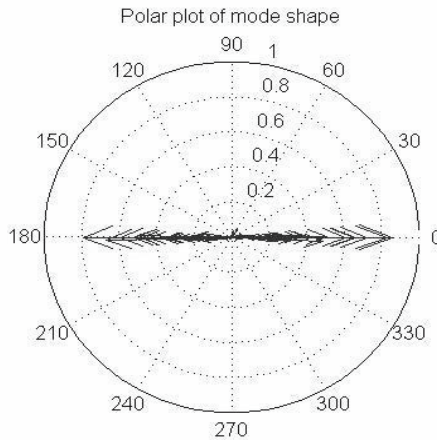


Fig. 12.10. Mode shape of first inter-area mode

The relation between the physical state and modal variables provides an important insight. Any arbitrary element v_{ki} in V can be seen as contribution of the i^{th} mode in the k^{th} state variable, i.e. activity of the i^{th} mode in the k^{th} state variable. On the other hand w_{ik} corresponds to a weighted contribution of the k^{th} state variable to i^{th} mode. The product $w_{ki}v_{ik}$ is, however, a dimensionless measure known as participation factor [12]. Both V and W can be assumed to be orthogonal and they can be scaled suitably such that $w_k v_i = 1.0$. The conditions in (12.64) and (12.65) would still be satisfied. The more generic definition of participation factor [5] is given as:

$$P_{ki} = \frac{|v_{ik}| |w_{ki}|}{\sum_{k=1}^{k=n} |v_{ik}| |w_{ki}|} \quad (12.73)$$

The participation factors for all of the fifteen complex modes are computed and shown in Table 12.7

Table 12.7. Normalized Participation factors

Eigenvalues (λ_s)	Normalized participation factors	Machine
-0.0626+j2.4212	0.10,0.06,0.05,0.03,0.03,0.03,0.026	13,15,16,9,14,6,3
-0.0697+j3.1469	0.24,0.23,0.02	16,14,15
-0.0507+j3.9564	0.25,0.04,0.03,0.03,0.03,0.027	13,9,6,12,5
-0.0810+j4.9710	0.32,0.13,0.04	15,14,16
-0.0927+j6.1810	0.38,0.03,0.02	9,5,6
-0.0684+j6.7719	0.16,0.15,0.06,0.034,0.03,0.02	2,3,5,6,4,7
-0.0387+j7.2178	0.38,0.07	12,13
-0.0568+j7.6901	0.20,0.14,0.09,0.05	5,6,7,4
-0.0752+j7.9630	0.25,0.24	2,3
-0.0542+j7.9915	0.20,0.10,0.08,0.02	10,1,8,12
-0.0537+j8.4600	0.24,0.12,0.11	10,8,1
-0.0617+j9.7802	0.22,0.20,0.03,0.025	8,1,7,6
-0.0716+j9.7820	0.21,0.17,0.03,0.03,0.03	7,6,4,8,1
-0.0701+j9.8349	0.30,0.12,0.05	4,5,7
-0.1161+j11.467	0.47	11

The participation factors for angle and speed for a particular machine are the same. We arrange them in descending order and show a few of them. The entries in the last column show the corresponding machines. It is seen that in low frequency electromechanical modes machines from different areas participate. These are known as inter-area modes [4]. At relatively high frequencies (>1.0 Hz), significant contribution is from one or two machines in a power plant. These are known as local and or intra-plant modes [4]. This is very useful when studying the behaviour of one machine with respect to the rest of the system. Participation factors reveal vital information for controlling low frequency oscillations of a system. Machines with higher participations are very effective for dampening oscillations and hence are the candidate machines to equip with power system stabiliser (PSS). For inter-area modes, PSS for many machines are needed and hence control de-

sign becomes a co-ordinated multivariable control design problem. One PSS is theoretically sufficient to control a local mode and obviously it is placed in the machine having the highest participation factor.

12.3 Modal Controllability, Observability and Residue

One drawback of the participation factor approach described in the previous section is that it only deals with the states and so it does not consider input and output parameters. It can not effectively identify controller site and appropriate feedback signal in the absence of information on input and output, which is more important when output feedback is employed. The effectiveness of control can, however, be indicated through controllability and observability indices. This is important as control cost is influenced to a great deal by the controllability and observability of the plant. These issues are addressed through modal controllability, observability and residue. This is briefly described next. The transfer function equivalent of equations (12.58) and (12.59) is:

$$G = C (sI - A)^{-1} B + D \quad (12.74)$$

Let us drop the direct transmission term D , as it does not influence the mode (exclusion does not affect our conclusion but simplifies the explanation) and re-write the first part of the right hand side as $G_r(s)$. We also make use of the orthogonal relationship between V and W i.e. $VW = I$:

$$\begin{aligned} G_r(s) &= C (sI - A)^{-1} B \\ &= CVW (sI - A)^{-1} VWB \\ &= CV [V^{-1} (sI - A) W^{-1}]^{-1} WB \\ &= CV (sI - \Lambda)^{-1} WB \\ &= \sum_{i=1}^n \frac{Cv_i w_i B}{s - \lambda_i} \\ &= \sum_{i=1}^n \frac{R_i}{s - \lambda_i} \end{aligned} \quad (12.75)$$

R_i is known as the modal residue, being the product of modal observability Cv_i and modal controllability $w_i B$. It is seen from (12.75) that modes with poor damping i.e. λ_i with small absolute real part, will significantly influence the magnitude of the transfer function G_r if it is scaled up by the residue R_i at and around the frequency corresponding to the imaginary part of λ_i . This means that controllability of the input signal and observability of the feedback signal become very important. The choice of feedback signal should be made after careful consideration. The feedback signal must have a high degree of sensitivity at and around the swing mode to be damped out. This will show as a high peak in the bode diagram. In other words, this means that the swing mode must be observable in the feedback

signal. The output signal must have little or no sensitivity to other swing modes. This is an obvious expectation from the perspective of minimum interaction amongst modes through the controller. A FACTS-device in a transmission line will only influence those modes responsible for power swings observed on that line. The expensive control effort could be wasted if it responds to local swings within an area at one end of the line. The effect of the feed forward term on the output is also very important. The feedback signal should have little or no sensitivity to its own output in the absence of a power swing. This is known as inner loop sensitivity [13] and does not involve swing mode dynamics. It results from feed forward effect of a signal by-passing the swing mode loop. In single-input-single-output (SISO) design, the output matrix C and input matrix B in equation (12.75) are row and column vectors respectively and hence the residue would be a complex scalar. As the residue is a complex variable, both magnitude and phase become important. The higher magnitude of the residue implies reduced control effort (gain) whereas higher phase lag requires multiple phase compensation blocks in the feedback path.

The FACTS-devices are never sited in a location of the system with highest modal controllability. Steady state power flow and dynamic voltage support dictate the criteria for locating. Nevertheless, if the devices are smaller in size and the sole purpose of having them installed in the system is to enhance small signal stability margin, the modal controllability can be used to find the most effective location. The modal controllability index ($w_i B$) at bus location (shunt device) or line (series device) can provide valuable information about the potential locations. We have computed the modal controllability indices of an SVC in all bus locations of the study system, which is further normalized with respect to the highest modal controllability vector. The absolute values are displayed in Table 12.8.

Table 12.8. Normalized modal controllability indices at various bus locations

bus location	cont indices	bus location	cont indices	bus location	cont indices	bus location	cont indices
01	0.37	18	1.00	35	0.11	52	0.75
02	0.58	19	0.09	36	0.17	53	0.27
03	0.62	20	0.97	37	0.79	54	0.52
04	0.85	21	0.93	38	0.18	55	0.67
05	0.99	22	0.95	39	0.11	56	0.69
06	0.89	23	0.95	40	0.11	57	0.65
07	0.89	24	0.92	41	0.03	58	0.66
08	0.54	25	0.56	42	0.04	59	0.62
09	0.20	26	0.58	43	0.12	60	0.60
10	0.13	27	0.68	44	0.12	61	0.25
11	0.10	28	0.38	45	0.05	62	0.70
12	0.13	29	0.30	46	0.12	63	0.69
13	0.06	30	0.25	47	0.23	64	0.78
14	0.02	31	0.22	48	0.19	65	0.71
15	0.03	32	0.16	49	0.06	66	0.73
16	0.05	33	0.16	50	0.08	67	0.85
17	0.14	34	0.15	51	0.01	68	0.89

It is seen that bus 18 is the most effective location for SVC to offer effective damping of the first inter-area mode (0.39 Hz). This is for a particular power flow, specific to a type of load and network conditions. Changes in any of these conditions may produce a different bus location with the highest controllability. The modal observability (Cv_i) indices on the other hand relate to feedback signals. Once the location is selected, the modal controllability is fixed. One particular type of signal such as power or line current or speed can be taken and the modal observability indices would be computed. The comparison of modal observability of two types of signal such as bus voltage magnitude and power in a line must not be done. Even though they are expressed in p.u., one p.u. voltage does not necessarily ensure one p.u. of line power. We have computed modal observability of line real power for SVC located at bus 18. The 700 MW power with constant impedance load and full tie line strength was considered i.e. operating condition was similar to that used for computing the modal observability. The results are shown in Table 12.9.

It is seen that the active power in the line between bus 13 and bus 17 has the highest normalized modal observability magnitude (in this case 1.00). There are many other signals having high modal observability too. The results in Table 12.9 reveal an interesting fact. The power signal in the line 13-17 with modal observability of 1.00 is the most effective signal to dampen first inter-area mode with least control effort. However, the signal needs to be transmitted from a remote location, making it less reliable. The observability of power signals in the lines originating from bus 18 are 0.14 (line 18-42), 0.30 (line 18-50) and 0.41 (line 16-18). These are local signals and so they are reliable, but need higher control effort when compared to remote signals. Synchronised phasor measurement technology coupled with dedicated high speed fibre optic network is available to use remote signal for damping of oscillations [7].

Table 12.9. Normalized modal indices for various line power signal

line between	obsrb indices	line between	obsrb indices	line between	obsrb indices	line between	obsrb indices
13-17	1.00	18-49	0.32	25-54	0.18	56-57	0.12
45-51	0.72	15-42	0.32	66-67	0.17	21-68	0.12
50-51	0.60	31-38	0.31	12-36	0.17	39-44	0.12
34-35	0.59	43-44	0.31	30-61	0.17	19-20	0.12
35-45	0.58	17-43	0.30	59-60	0.17	31-53	0.12
18-50	0.56	18-50	0.30	58-59	0.16	56-66	0.11
34-36	0.52	37-68	0.28	67-68	0.16	21-22	0.10
47-53	0.52	47-48	0.24	58-63	0.15	06-22	0.09
53-54	0.49	14-41	0.21	62-63	0.14	24-68	0.09
40-41	0.44	54-55	0.21	30-31	0.14	01-54	0.09
40-48	0.43	52-55	0.20	18-42	0.14	27-37	0.09
16-18	0.41	37-52	0.20	44-45	0.14	30-32	0.09
38-46	0.38	19-68	0.19	32-33	0.13	03-62	0.09
60-61	0.37	37-52	0.19	30-53	0.13	09-29	0.09
17-36	0.37	41-42	0.19	25-26	0.13	30-32	0.09
46-49	0.35	57-60	0.19	21-68	0.12	09-29	0.09

In this chapter, the modelling of various components of power systems is discussed. A power injection and small signal model of various FACTS-devices are described. The eigenvalue analysis on a 16-machine and 68-bus study system model is carried out using various modelling complexities. The influence of various system operating conditions on the damping and frequencies of electromechanical modes are analysed. It is concluded that power flow, load characteristics and network topologies affect the damping and frequency of inter-area mode significantly. The method of participation factor is applied to compute relative participation of machine in a particular mode to identify most effective machine for installing power system stabilizer. The methods of modal controllability and observability are explained. The modal controllability indices of a static var compensator at various bus locations of the study system are computed as an effective approach to identify best location. The computed modal observability indices in power signals from various lines provide useful information on the most effective stabilising signal.

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